Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl16

Electrohydrodynamic Instability in Nematics—Effect of Sample Thickness and Magnetic Field

Papiya Nandy [a]
[a] Physics Department, Jadavpur University, Calcutta, 700 032, India
Version of record first published: 20 Apr 2011.

Mol. Cryst. Liq. Cryst., 1984, Vol. 104, pp. 281–288
0026-8941/84/1044-0281/$18.50/0

Electrohydrodynamic Instability in Nematics — Effect of Sample Thickness and Magnetic Field

PAPIYA NANDY

Physics Department, Jadavpur University, Calcutta 700 032. India

(*Received July 5, 1983*)

The onset of electrohydrodynamic instability in an uniformly aligned nematic liquid crystal under a steady magnetic field and a low frequency electric field has been theoretically investigated, taking into account the boundary conditions. The result shows that the threshold voltage for the onset of instability and the ratio of wavelength of Williams domain to the sample thickness are functions of the product of the magnetic field intensity and the sample thickness, defined here as β, as well as on the direction of the applied magnetic field.

INTRODUCTION

The criteria for the onset of instability induced by electric and magnetic fields in nematic liquid crystals had been studied by several authors.[1-6] The first successful theoretical treatment of the effect of a periodic disturbance in a nematic subjected to a *dc* field was given by Helfrich.[1] Inclusion of the boundary conditions in the original one-dimensional treatment of Helfrich was due to Ford and Penz.[2] Barnik *et al.* had shown that for homeotropic orientation, Helfrich's model does not agree with experiments even qualitatively, whereas the discrepancy is quantitative for planar orientations.[3] Peggi and Fillipini in a study of the effect of a magnetic field on the order parameter, found that the birefringence induced by the magnetic field varies linearly with the field and this was later satisfactorily explained by Malraison *et al.*[4,5] Laidlaw had theoretically established the conditions for oscillatory instabilities of nematic liquid crystals in steady electric and magnetic fields.[6]

In this paper we have studied the influence of sample thickness d and an externally applied magnetic field B on the low frequency threshold voltage V_{th} and wavelength of disturbance in a nematic liquid crystal. Our calculation shows that (a) V_{th} is a function of the product of B and d, defined here as β[7], (b) instability can be obtained by applying only a magnetic field perpendicular to the director orientation in a planar conformation and (c) the ratio of period of William's domain to the sample thickness, λ/d, is a function of β and of the material constants. This ratio was taken to be 2, same for all materials, by Helfrich,[1] for an approximate treatment of the boundary effects. In our case, the ratio approaches 1.5 for $\beta = 0$, the physical situation corresponding to zero magnetic field.

BASIC EQUATIONS

In a planar conformation, the director $\vec{n}$ of the unperturbed nematic is assumed to be parallel to the x-axis of a Cartesian coordinate system and a low frequency ac electric field is applied along the z-axis. The magnetic field can be applied either along the x-axis, in which case it stabilizes the system or it can be applied along the z-axis when it helps to bring the instability. We investigate the criteria for instability in a sample of nematic liquid crystal in this set up, subject to the boundary conditions.

The starting equations are the torque and force balance equations along with the charge conservation equation.[8] A detailed derivation was given by Ford and Penz[2] and so we emphasize only the part which is an extension of their work. In this treatment only bend modes of small amplitudes are considered and the fluid is assumed to be incompressible. The different vector fields in the first order approximation are as follows:

$$\vec{n}(t) = (1, 0, \theta_1(t)\exp i\vec{q}\cdot\vec{r})$$

$$\vec{v}(t) = (-s, 0, 1)v_1(t)\exp i\vec{q}\cdot\vec{r}$$

$$\vec{E}(t) = (E_1(t)\exp i\vec{q}\cdot\vec{r}, 0, E_0(t) + sE_1(t)\exp i\vec{q}\cdot\vec{r})$$

$$\vec{H}(t) = (H_0 + H_1(t)\exp i\vec{q}\cdot\vec{r}, 0, sH_1(t)\exp i\vec{q}\cdot\vec{r})$$

where $s = q_z/q_x$; $\vec{v}$ is the hydrodynamic velocity; E_0 and H_0 are the applied electric and magnetic fields respectively; E_1 and H_1 are the fields induced due to the distortion in the nematic film and are much small compared to E_0 and H_0. This expression for $\vec{H}$ is compatible with the steady state Maxwell's equation: $\vec{\nabla} \times \vec{H} = 0$.

The magnetic induction $\vec{B}$, in terms of magnetic permeability tensor $\vec{\mu}$ is:

$$\vec{B} = \vec{\mu}\vec{H} = \mu_0(1 + \vec{\chi})\vec{H}$$

where
$$\mu_0 = 4\pi \times 10^{-7} \text{ newtons/amp}^2$$
$$\chi_{ij} = \chi_2\delta_{ij} + \chi_a n_i n_j$$

χ_a being the diamagnetic susceptibility anisotropy and is small in magnitude ($\sim 10^{-6}$). To satisfy Maxwell's equation for magnetic induction, namely, $\operatorname{div}\vec{B} = 0$, H_1 must take the form,

$$H_1 = -s\chi_a H_0\theta_1/(1 + s^2)$$

The force and the torque balance equations are:

$$-ip_1 + \{\eta_2 s^2 + (\eta_2 + \alpha_5)\} q_z v_1 + is\alpha_3\dot{\theta}_1 = 0 \qquad \text{(i)}$$

$$-isp_1 + [(\alpha_2 - \alpha_4 + \alpha_5)s^2/2 - \eta_1] q_x v_1 + i\alpha_2\dot{\theta}_1$$
$$+ i\epsilon_0\epsilon_a E_0^2\theta_1 + i\epsilon_0 E_0(\epsilon_1 + s^2\epsilon_2)E_1 = 0 \qquad \text{(ii)}$$

$$i(\alpha_2 - s^2\alpha_3)q_x v_1 + q_x^2\theta_1[k_{33} + s^2 k_{11}] + \mu_0\chi_a H_0^2\theta_1$$
$$-\epsilon_0\epsilon_a E_0^2\theta_1 - \epsilon_0\epsilon_a E_0 E_1 + \gamma_1\dot{\theta}_1 = 0 \qquad \text{(iii)}$$

and the charge conservation equation is:

$$\sigma_a E_0\theta_1 + E_1(\sigma_1 + s^2\sigma_2) = 0 \qquad \text{(iv)}$$

where α's, γ's and η's are the viscosity coefficients, k's are the elastic constants, σ and ϵ are the conductivity and dielectric constants of the medium (1-direction $\parallel$ and 2-direction $\perp \vec{n}$); p_1 is the amplitude of deviation from p_0, the uniform pressure. Let the magnetic field be applied along x-axis. (When the field is applied along z-direction, $\chi_a H^2$ has a negative sign in eqn. (iii). For aromatic compounds, $\chi_a > 0$ and hence in the first case the field stabilizes the original alignment, whereas in the second case it helps to bring the instability.)

On the assumption of a sinusoidal electric field:

$$E_0(t) = E_M \cos wt$$

the other time dependent fields can take the form:

$$\theta_1(t) = \sum_n a_{1,n} \exp inwt; \quad v_1(t) = \sum_n a_{2,n} \exp inwt$$

In the low frequency limit, the terms containing the time derivatives in eqns. (i) to (iii) vanish and we get a time-averaged equation:

$$\begin{aligned}(1+s^2)\{(\eta_1+\eta_2 s^2)\{-(\sigma_1/\sigma_2+s^2)\\ \times\left[(k_{33}+s^2k_{11})+\mu_0\chi_a H_0^2/q_x^2\right]\\ +(1+s^2)\epsilon_0\epsilon_a E_{\text{th}}^2/2q_x^2\}\\ +(\alpha_2-s^2\alpha_3)(\epsilon_1-\epsilon_2\sigma_1/\sigma_2)E_{\text{th}}^2/2q_x^2\} = 0 \qquad \text{(v)}\end{aligned}$$

where E_{th} is the threshold field along z-direction.

This is an eighth order equation in s, which reduces to Ford and Penz's result[2] for $H_0 = 0$ and to Helfrich's result[1] for $s = 0$. This gives a relation between the threshold field E_{th} and s in terms of the material constants in an infinite medium. The boundary conditions are introduced at this point.[2] They are: at the plates ($z = \pm d/2$), the z-components of $\vec{v}$ and $\vec{n}$ and the x-components of $\vec{E}$ and $\vec{v}$ vanish. Simultaneous solutions of these four boundary conditions is possible under the condition:

$$\begin{vmatrix} \cos s_1\phi & \cos s_2\phi & \cos s_3\phi & \cos s_4\phi \\ s_1\sin s_1\phi & s_2\sin s_2\phi & s_3\sin s_3\phi & s_4\sin s_4\phi \\ M_1\cos s_1\phi & M_2\cos s_2\phi & M_3\cos s_3\phi & M_4\cos s_4\phi \\ M_1N_1\cos s_1\phi & M_2N_2\cos s_2\phi & M_3N_3\cos s_3\phi & M_4N_4\cos s_4\phi \end{vmatrix} = 0 \qquad \text{(vi)}$$

where,

$$M_i = (\alpha_2/\alpha_3 - s_i^2)/\{(k_{33}+s_i^2k_{11})N_i - (1+s_i^2)\epsilon_0\epsilon_a V_{\text{th}}^2/8\phi^2 + N_i\mu_0\chi_a H^2d^2/4\phi^2\}$$

$$N_i = (\sigma_1/\sigma_2 + s_i^2)$$

$$\phi = dq_x/2 = \pi d/\lambda$$

V_{th} = threshold voltage required for the onset of instability across the sample of thickness d.

λ = wavelength of disturbance.

The above equations are derived when a magnetic field is applied parallel to director orientation. The same set of equations may be obtained when the magnetic field is applied perpendicular to director orientation along z-direction, except with a change of sign for the terms containing magnetic field.

RESULTS AND DISCUSSION

The two equations (v) and (vi) have been solved numerically with the help of Burrough's 5500 computer. Using the material constants typical for PAA at 120°C, the result of the numerical calculations gives the threshold voltage required for the onset of electrohydrodynamic modes for sample of varying thickness under variable magnetic field. When the magnetic field applied parallel to director orientation (stabilizing field) is increased, V_{th} increases. This is quite expected, as with the increase in the value of the stabilizing magnetic field, higher magnitude electric field torques will be necessary to bring the instability (Figure 1a). The situation is reversed when the magnetic field is perpendicular to $\vec{n}$; here the instability sets in for a much lower value of V_{th} (Figure 1b). Figure 2a shows the dependence of V_{th} on sample thickness in presence of a stabilizing magnetic field. In the absence of magnetic field, this dependence vanishes (Ford and Penz's result).[2] The effect of sample thickness on the wavelength of disturbance λ in the stabilizing field is shown in Figure 2b.

As a consequence of eqns. (v) and (vi), it can be shown that the two variables, namely the magnetic field and the sample thickness are equivalent in influencing the onset of instability in nematic liquid crystal under a low frequency electric field. More explicitly, λ/d and V_{th} depend only on the product of B and d. Thus a variable β defined as $\beta = B \cdot d$ may be introduced and Figure 3 shows the dependence of V_{th} and λ/d on β. For $\vec{B} \parallel \vec{n}$, as β decreases, V_{th} decreases and attains a value 8.1 volts for $\beta = 0$, which is the value of V_{th} in the absence of a magnetic field (result of Ford and Penz[2]). For $\vec{B} \perp \vec{n}$, V_{th} decreases when β increases, meaning that an electric field of smaller magnitude can start the instability either for large sample thickness or for higher magnetic field intensity. The reverse result is true for the dependence of λ/d on β. For $\vec{B} \parallel \vec{n}$, λ/d decreases for an increase of β and the situation reverses for $\vec{B} \perp \vec{n}$. For $\beta = 0$, i.e. the physical situation corresponding to zero magnetic field, λ/d approaches the value ~ 1.5. The value of this ratio was taken to be 2 in Helfrich's approximate treatment of the boundary effects and was independent of the material

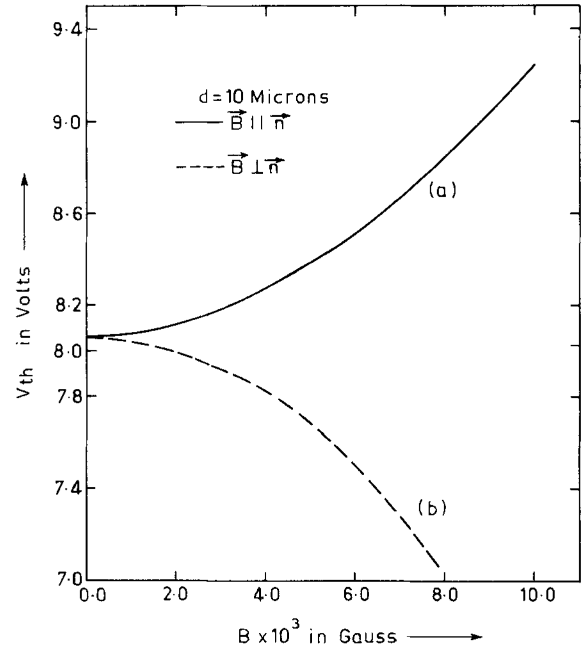

FIGURE 1 Threshold voltage V_{th} as a function of B. (a) $\vec{B} \parallel \vec{n}$; (b) $\vec{B} \perp \vec{n}$. Sample thickness = 10 micron.

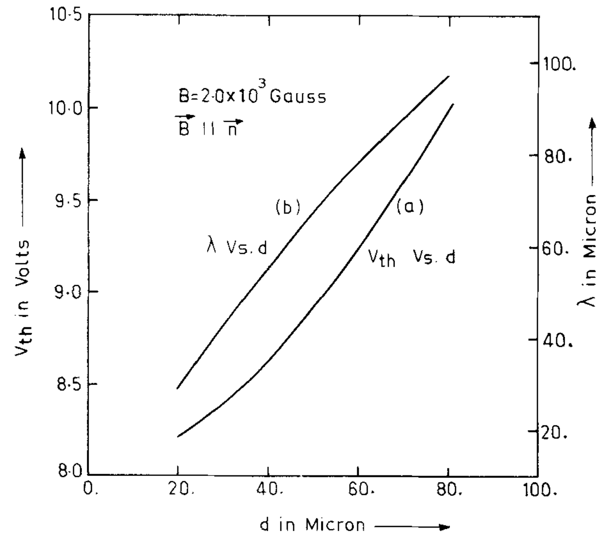

FIGURE 2 Effect of sample thickness on (a) threshold voltage V_{th}; (b) wavelength of disturbance λ. Intensity of stabilizing magnetic field = 2.0×10^3 Gauss.

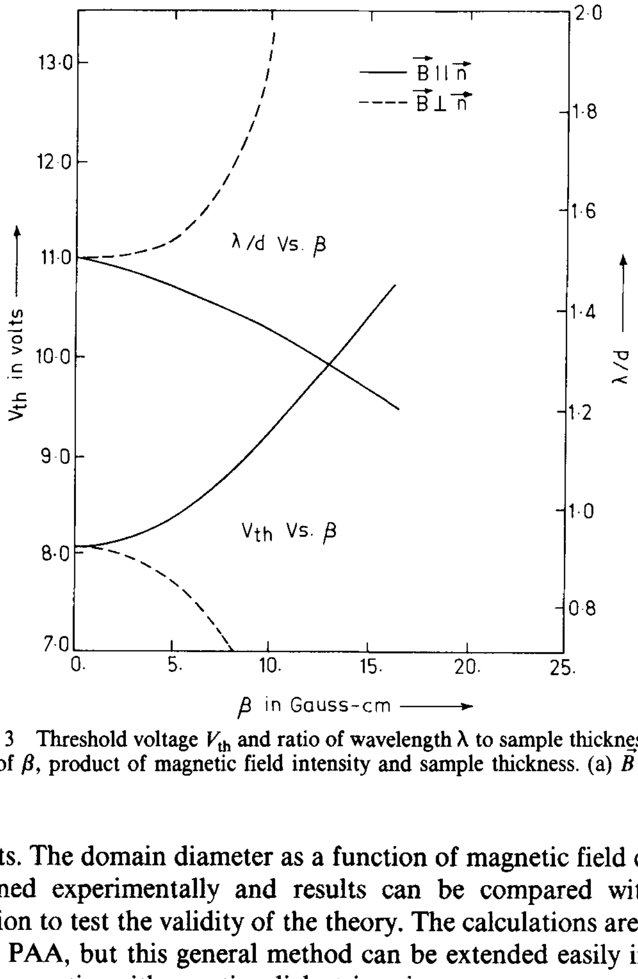

FIGURE 3 Threshold voltage V_{th} and ratio of wavelength λ to sample thickness d as a function of β, product of magnetic field intensity and sample thickness. (a) $\vec{B} \parallel \vec{n}$; (b) $\vec{B} \perp \vec{n}$.

constants. The domain diameter as a function of magnetic field can be determined experimentally and results can be compared with the calculation to test the validity of the theory. The calculations are made here for PAA, but this general method can be extended easily in case of other nematics with negative dielectric anisotropy.

Acknowledgments

The author wishes to express her sincere gratitude to Prof. A. Saupe and late Prof. W. Franklin for the stimulating discussion and encouragement of this work.

References

1. W. Helfrich, *J. Chem. Phys.*, **51**, 4092 (1969).
2. P. A. Penz and G. W. Ford, *Phys. Rev. A*, **6**, 414 (1972).

3. M. I. Barnik, L. M. Blinov, M. F. Grebenkin, S. Pikin and V. G. Chigrinov, *Sov. Phys. JETP*, **69**, 1080 (1975).
4. Y. Poggi and J. C. Filippini, *Phys. Rev. Lett.*, **39**, 150 (1977).
5. B. Malraison, Y. Poggi and E. Guyon, *Phys. Rev. A*, **21**, 1012 (1980).
6. W. Laidlaw, *Phys. Rev. A*, **20**, 2188 (1979).
7. P. Sengupta, Ph.D. dissertation, Kent State University, Ohio 1973.
8. P. Sengupta and A. Saupe, *Phys. Rev. A*, **9**, 2698 (1974).